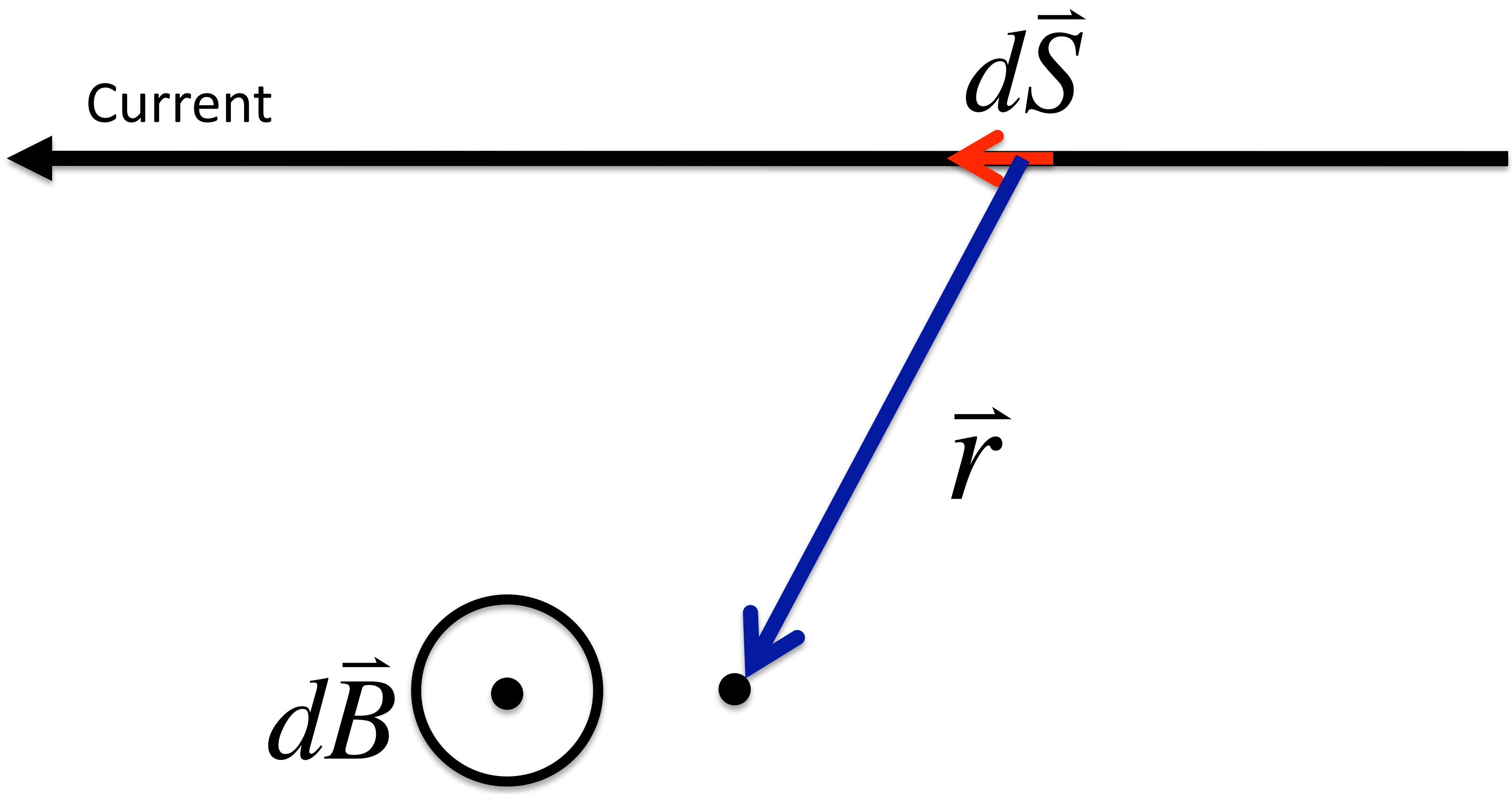
$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

Units of magnetic field: Tesla

1 T = 10000 Gauss

Surface of earth: 0.5×10^{-4} T
Bar magnet: 1.5 T
Strong superconducting magnet: 30 T

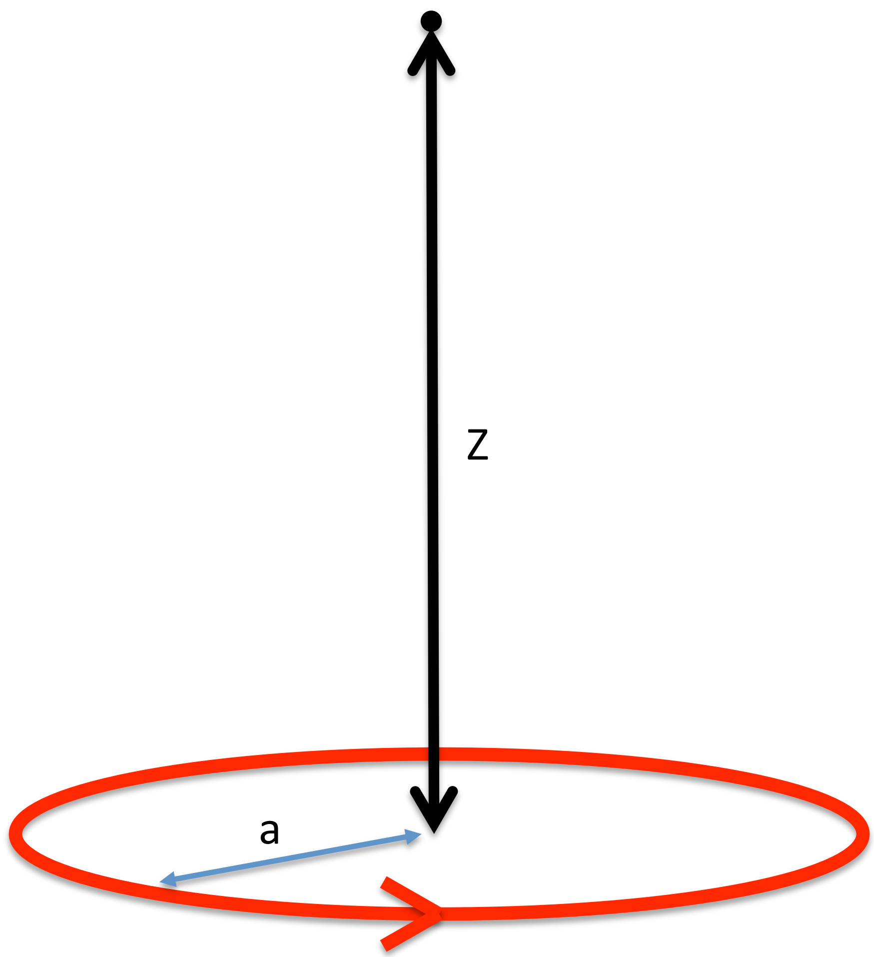
Biot-Savart Law



$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{S} \times \hat{r}}{r^2}$$

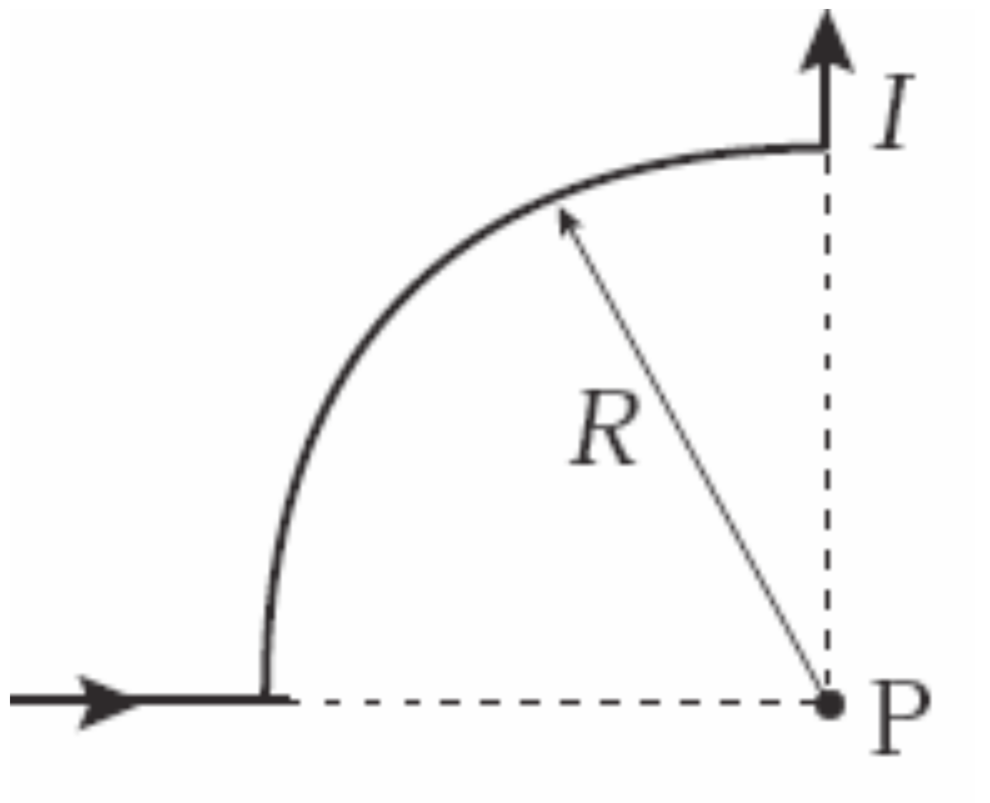
$$\mu_0 = 4\pi \times 10^{-7}\, T \bullet m / A$$

Infinite straight wire example



Z
a

The segment of wire (total length = 6R) is formed into the shape shown and carries a current *I*. What is the magnitude of the resulting magnetic field at the point P?



a. $\frac{\mu_0 I}{8R}$

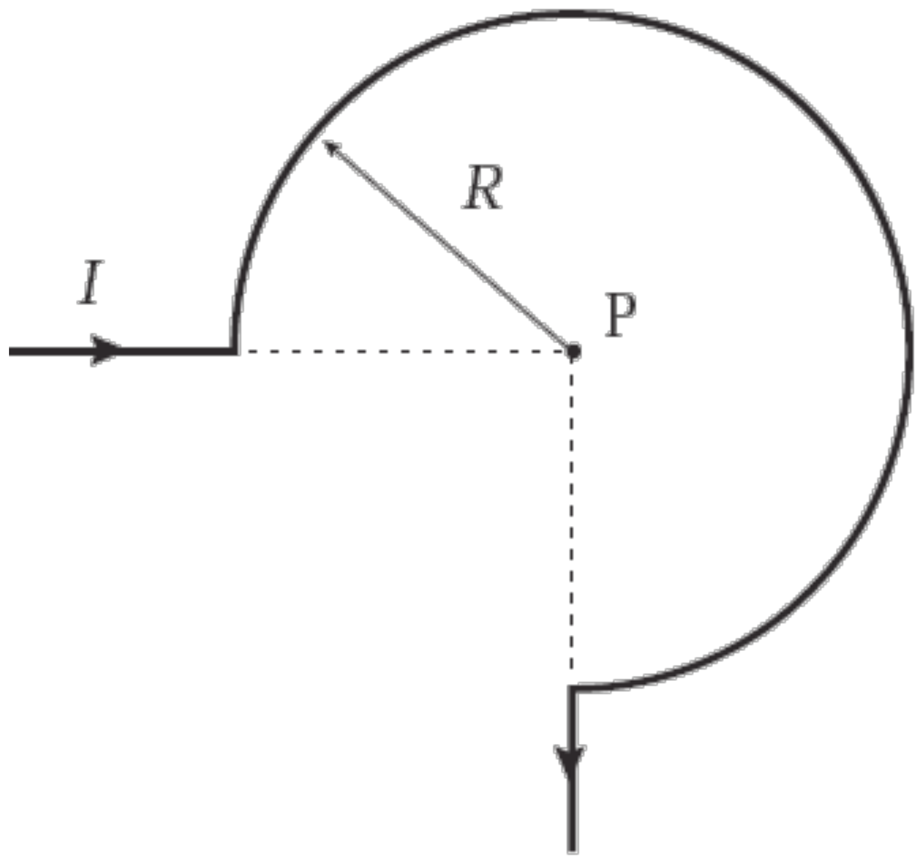
b. $\frac{\mu_0 I}{2R}$

c. $\frac{\mu_0 I}{4R}$

d. $\frac{\mu_0 I}{2\pi R}$

e. $\frac{\mu_0 \pi I}{8R}$

The segment of wire (total length = 6*R*) is formed into the shape shown and carries a current *I*. What is the magnitude of the resulting magnetic field at the point P?



a. $\dfrac{3\mu_0 I}{8R}$

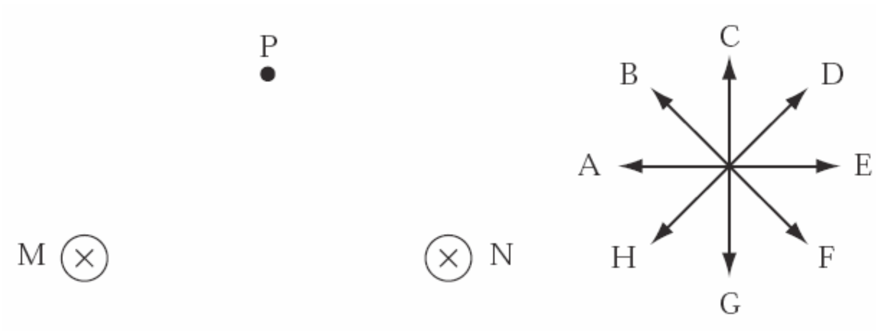
b. $\dfrac{3\mu_0 I}{2R}$

c. $\dfrac{3\mu_0 I}{4R}$

d. $\dfrac{3\mu_0 I}{2R}$

e. $\dfrac{3\mu_0 \pi \ I}{8R}$

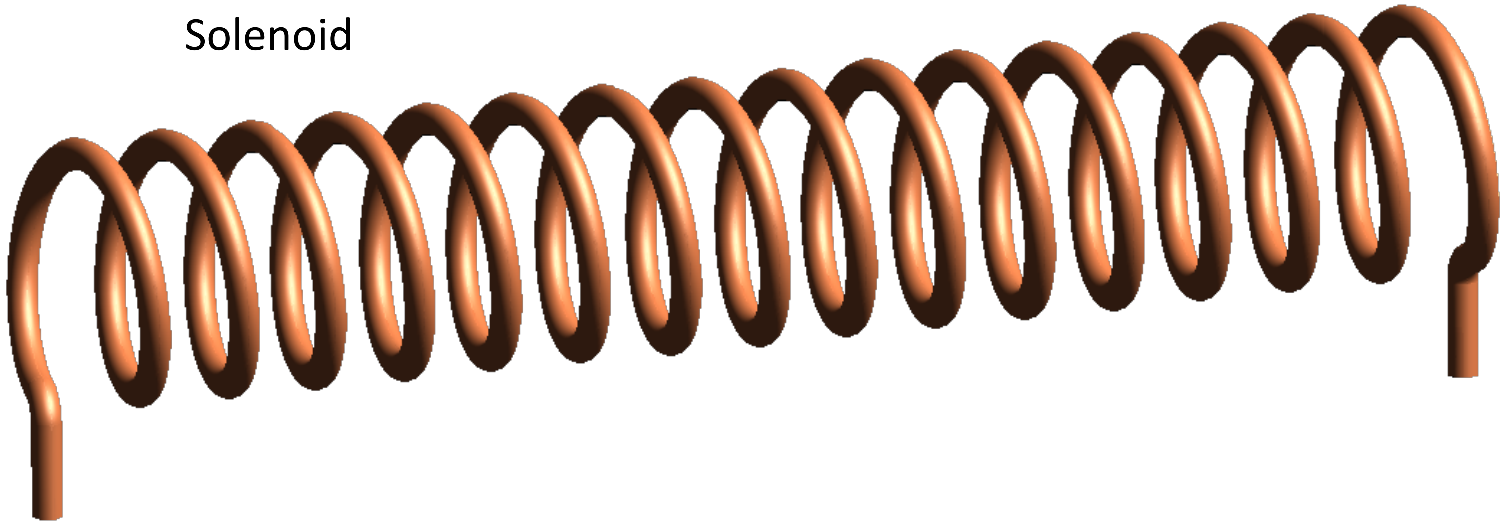
Question 1: Equal currents of magnitude *I* travel into the page in wires M and N. Eight directions are indicated by letters A through H.

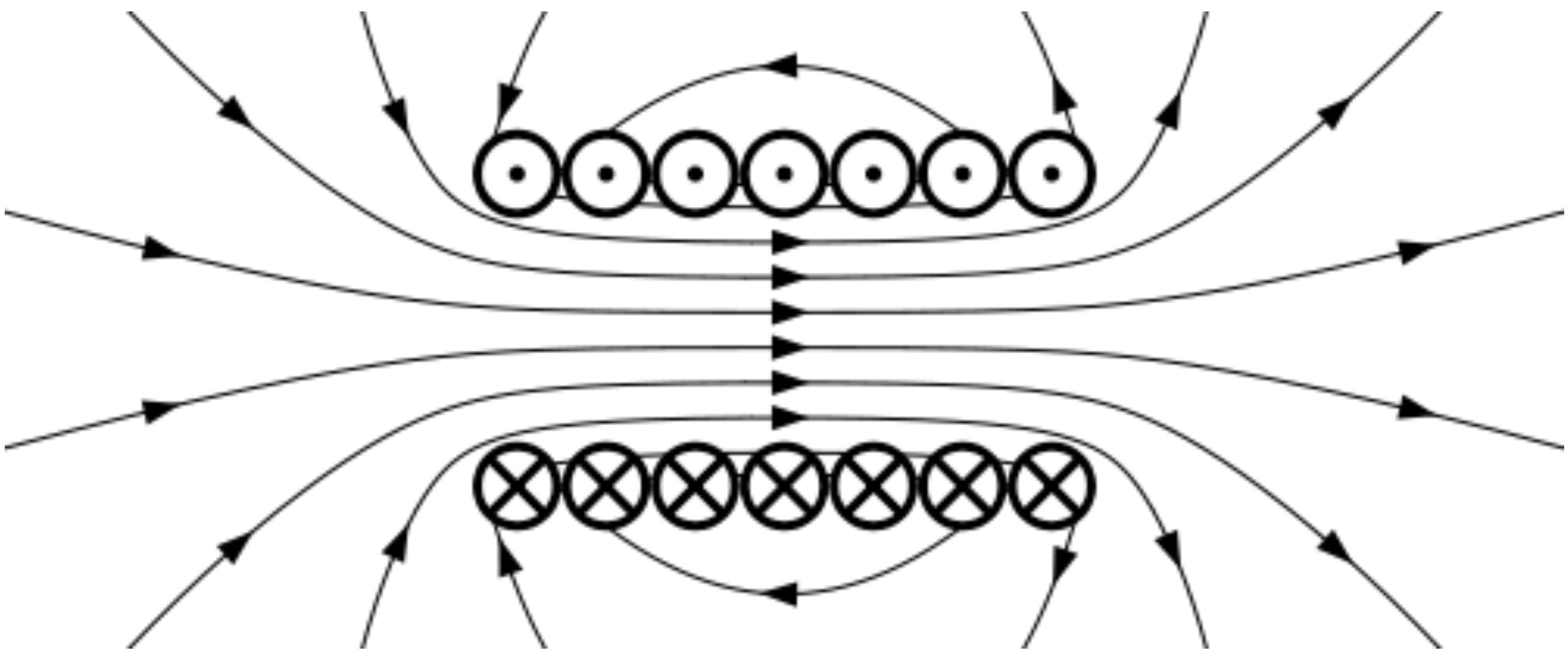


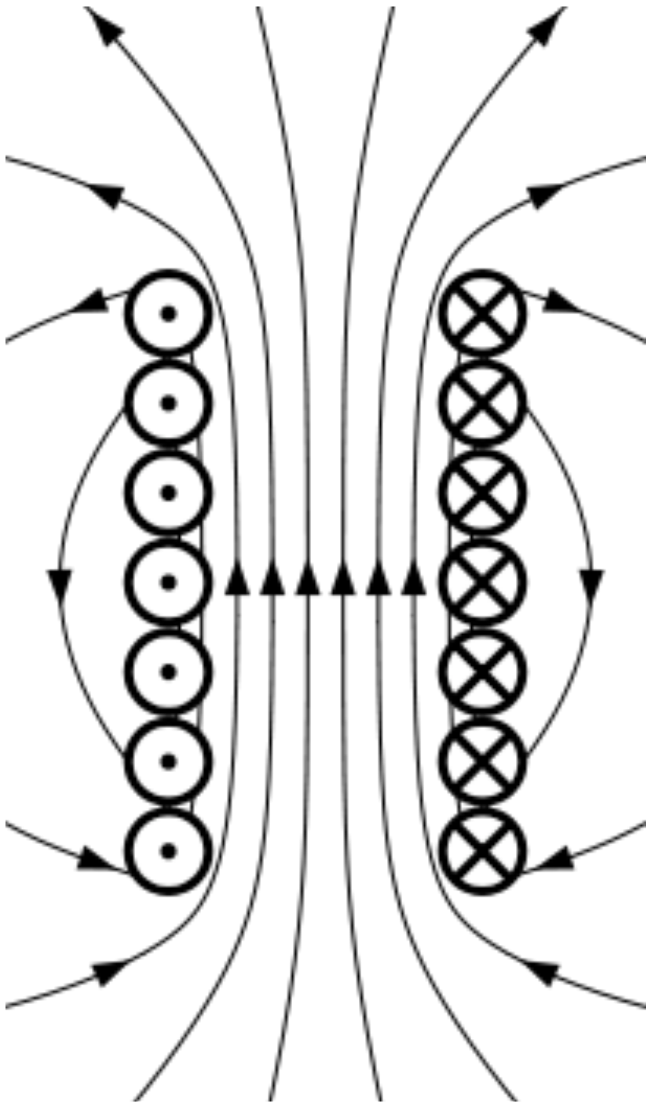
The direction of the magnetic field at point P is

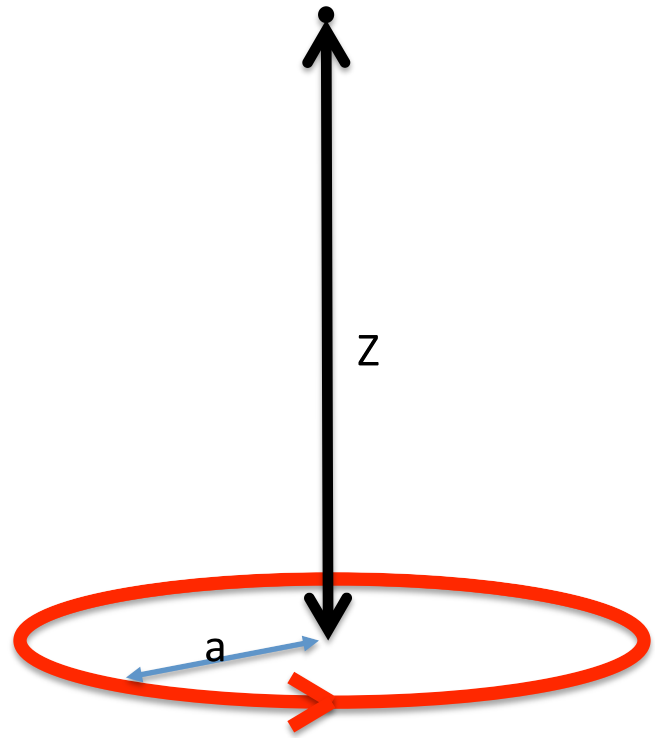
a. B.
b. C.
c. D.
d. E.
e. F.

Solenoid



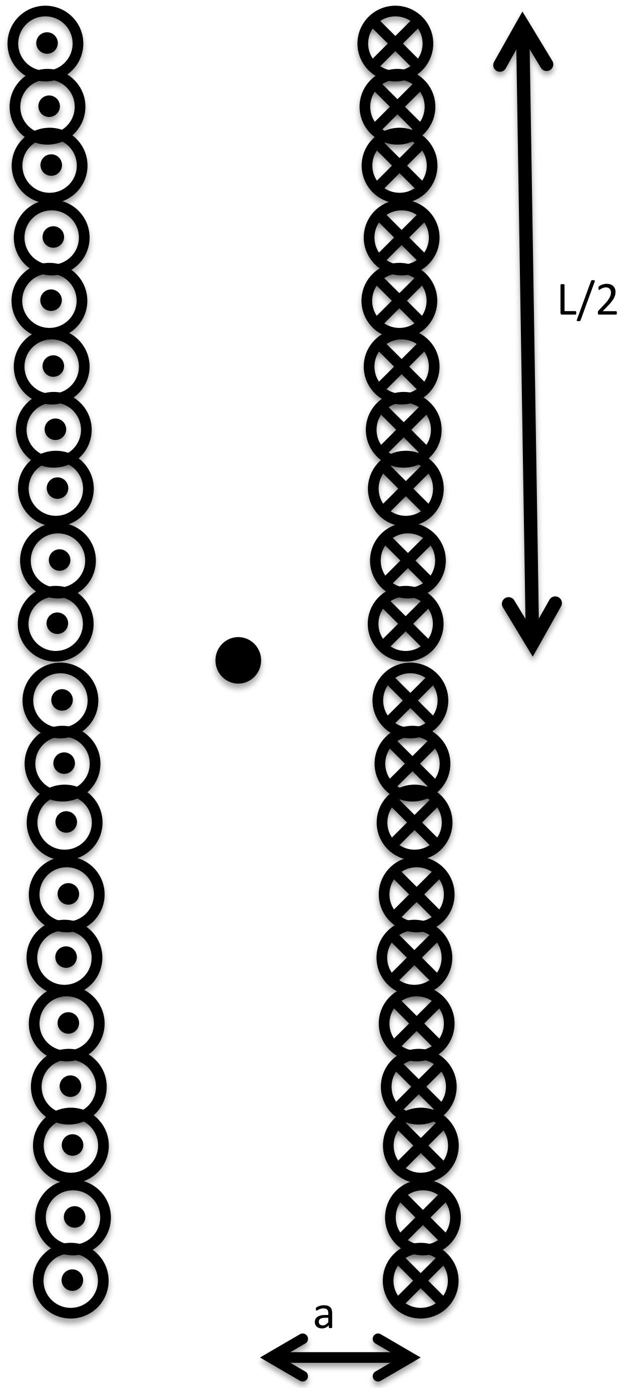






$$B = \frac{\mu_0 I}{2} \frac{a^2}{\left(a^2 + z^2\right)^{\frac{3}{2}}}$$

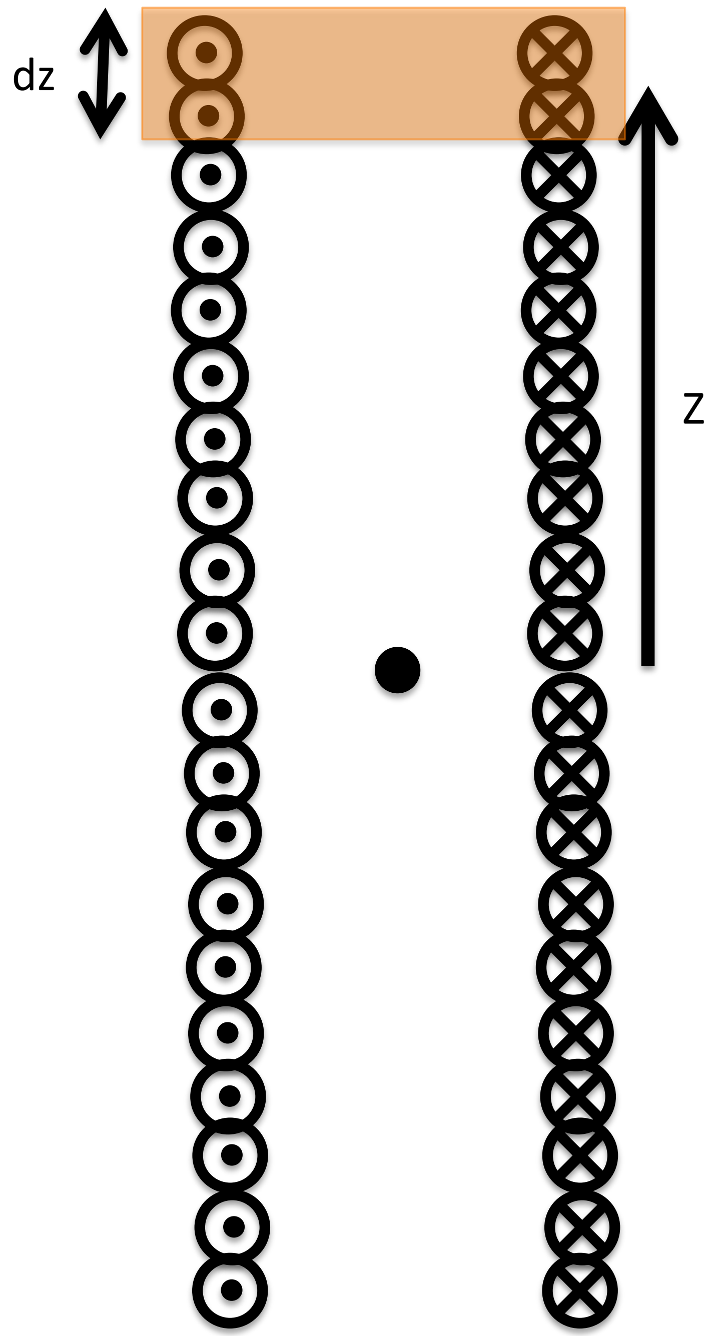
Approximate solenoid as bunch of separate loops



Magnetic field at the center of the solenoid

L : length of solenoid
n: coil density (#/m)
a: radius of the coil

Looks like integration is required again, but how?

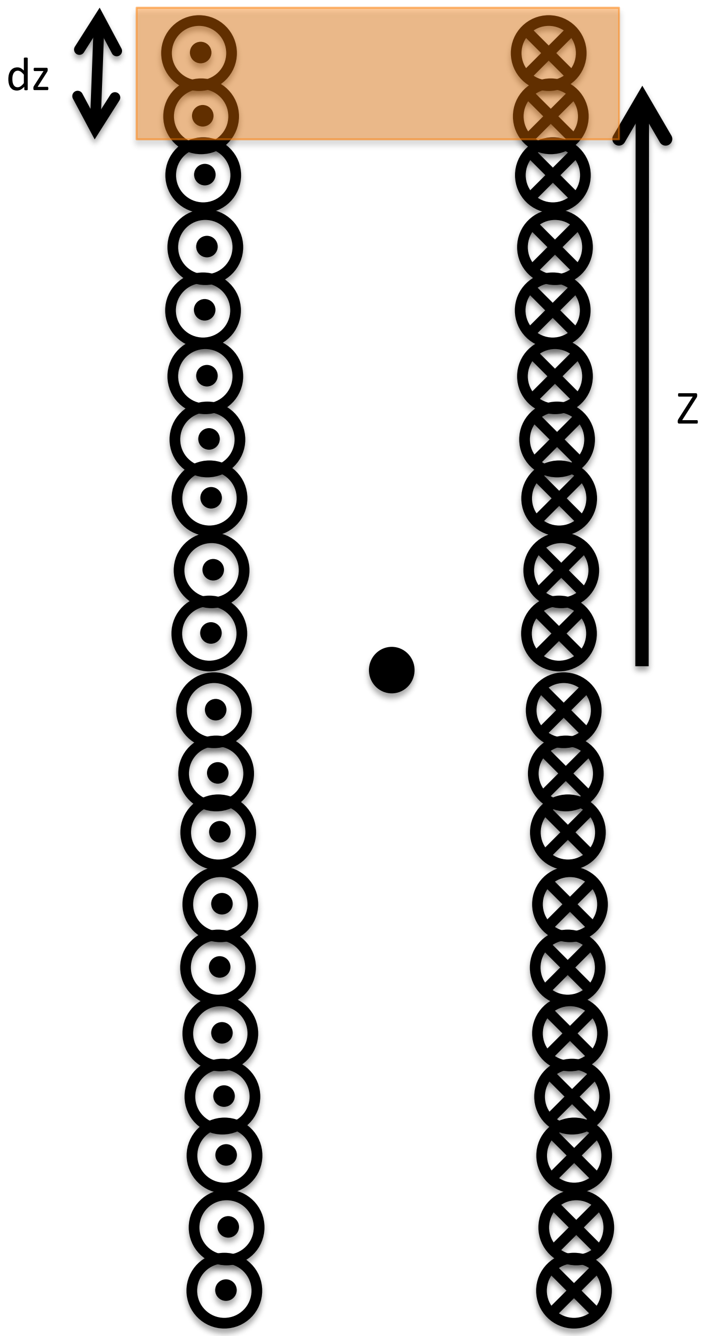


Current through each coil: I
Number of coils in dz: ndz

$$dB = n\frac{\mu_0 I}{2}\frac{a^2 dz}{\left(a^2+z^2\right)^{\frac{3}{2}}}$$

$$B = \int_{-L/2}^{L/2}\frac{\mu_0 nI}{2}\frac{a^2 dz}{\left(a^2+z^2\right)^{\frac{3}{2}}}$$

$$B = \frac{\mu_0 nIa^2}{2}\left[\frac{L/2}{a^2\left(a^2+\frac{L^2}{4}\right)^{\frac{1}{2}}} - \frac{-L/2}{a^2\left(a^2+\frac{L^2}{4}\right)^{\frac{1}{2}}}\right]$$



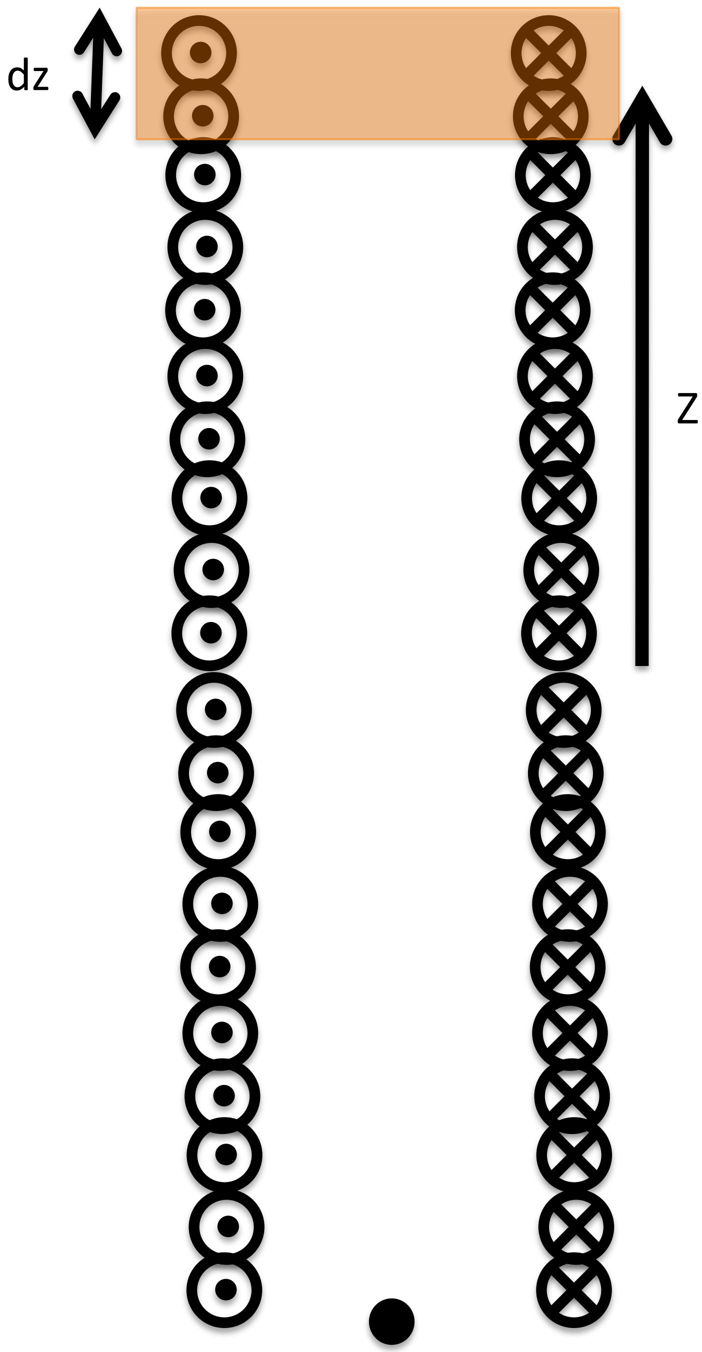
Current through each coil: I
Number of coils in dz: ndz

$$B = \frac{\mu_0 n I a^2}{2}\left[\frac{L/2}{a^2\left(a^2+\frac{L^2}{4}\right)^{\frac{1}{2}}} - \frac{-L/2}{a^2\left(a^2+\frac{L^2}{4}\right)^{\frac{1}{2}}}\right]$$

$$B = \frac{\mu_0 n I a^2}{2}\frac{L}{a^2\left(a^2+\frac{L^2}{4}\right)^{\frac{1}{2}}}$$

If L>>a

$$B \sim \frac{\mu_0 n I a^2}{2}\frac{L}{a^2\left(\frac{L^2}{4}\right)^{\frac{1}{2}}} = \frac{\mu_0 n I}{2}\frac{L}{L/2} = \mu_0 n I$$



What about the field at the very end?

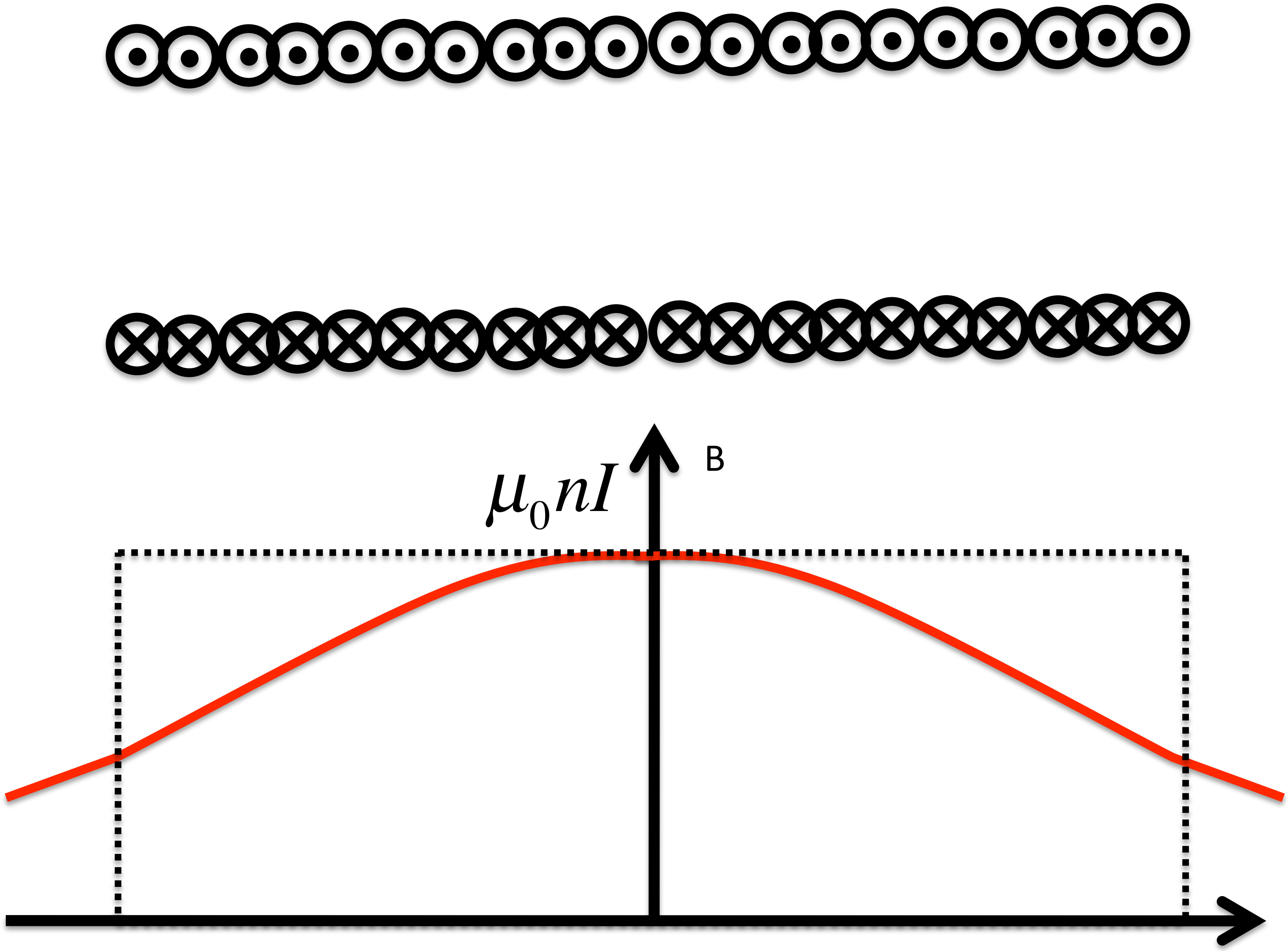
$$B = \int_0^L \frac{\mu_0 n I}{2} \frac{a^2 dz}{\left(a^2 + z^2\right)^{\frac{3}{2}}}$$

$$B = \frac{\mu_0 n I a^2}{2} \left[\frac{L}{a^2\left(a^2 + L^2\right)^{\frac{1}{2}}} - \frac{0}{a^2\left(a^2\right)^{\frac{1}{2}}} \right]$$

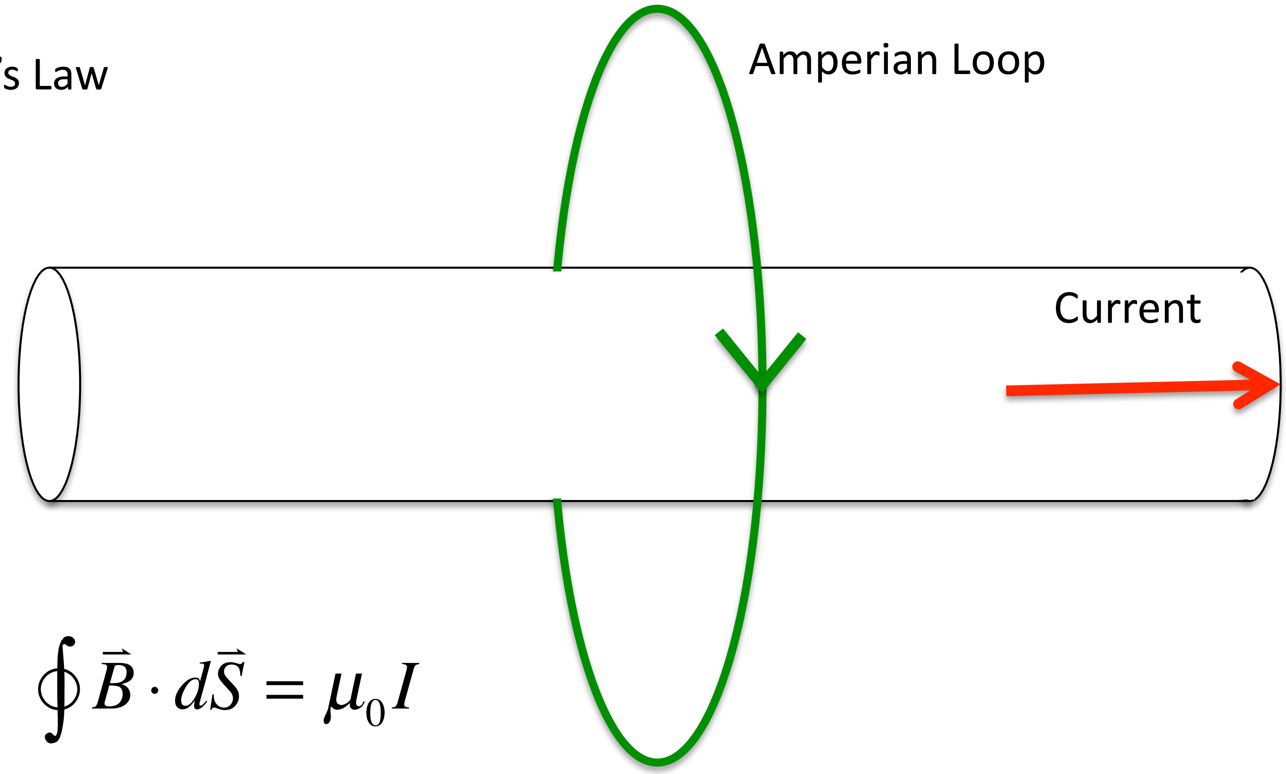
$$B = \frac{\mu_0 n I a^2}{2} \frac{L}{a^2(a^2 + L^2)^{1/2}}$$

If L>>a

$$B \sim \frac{\mu_0 n I a^2}{2} \frac{L}{a^2\left(L^2\right)^{\frac{1}{2}}} = \frac{\mu_0 n I}{2} \frac{L}{L} = \frac{\mu_0 n I}{2}$$


B
$\mu_0 nI$

Ampere's Law

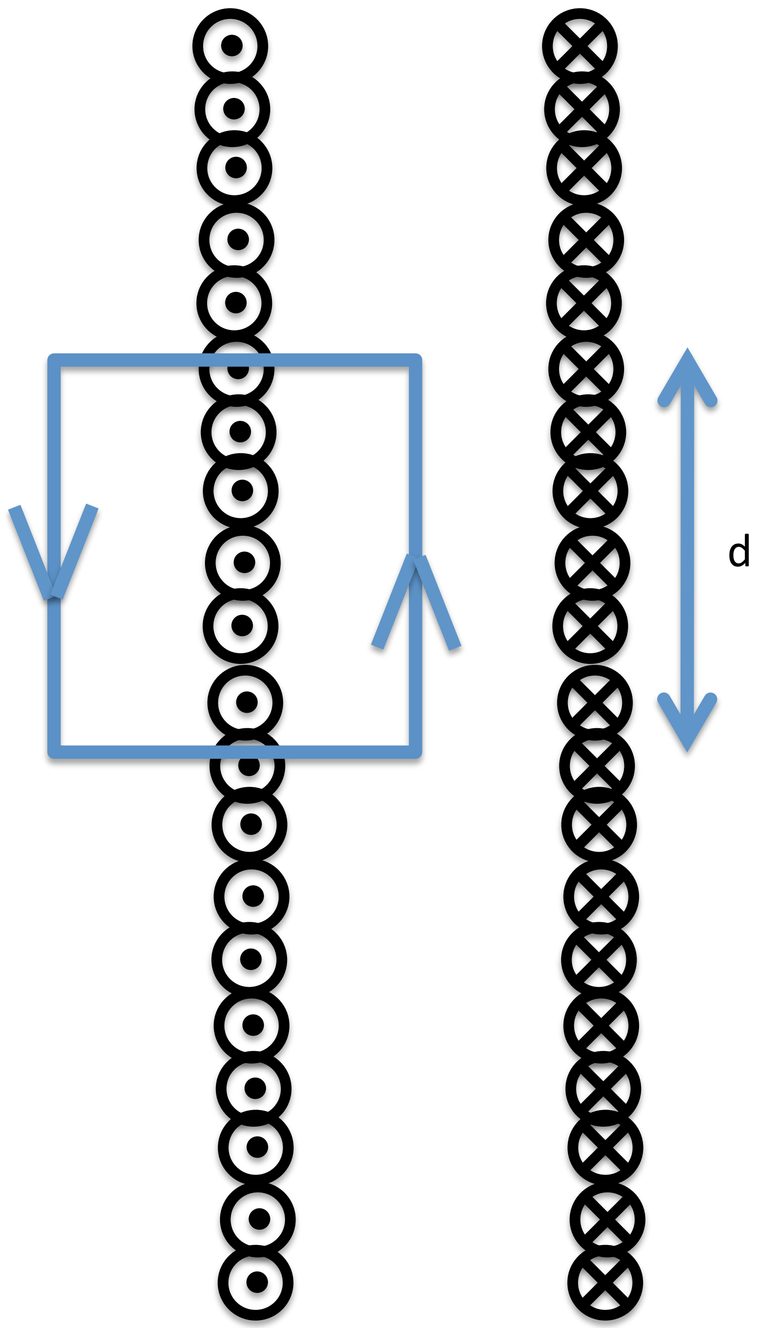


$$\oint \vec{B} \cdot d\vec{S} = \mu_0 I$$

Integral evaluated around any closed path where I is the total current passing through any surface defined by the path

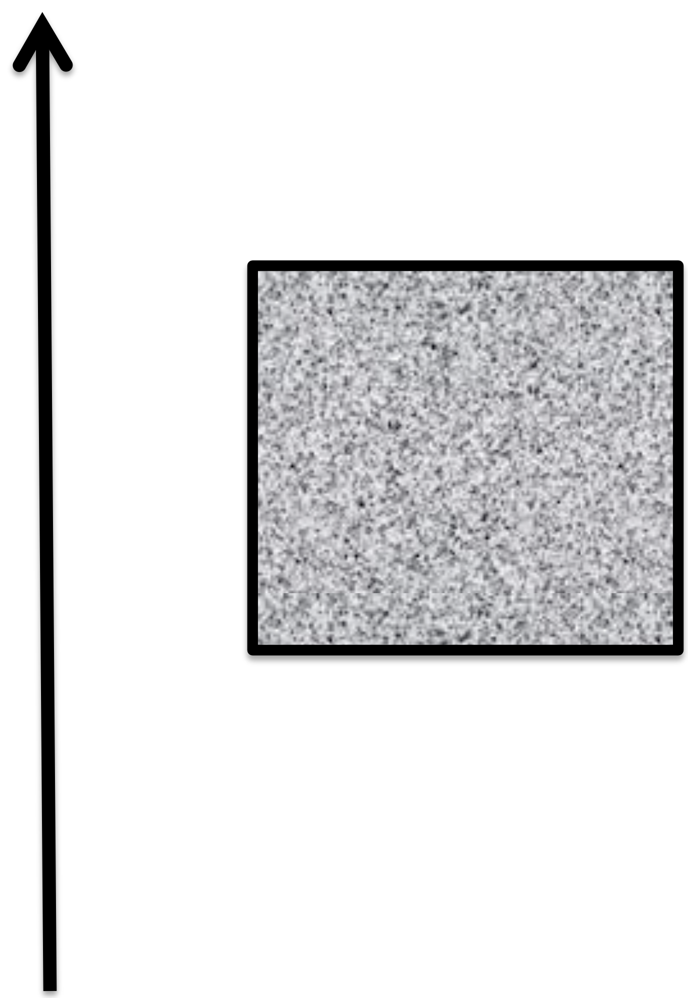
Example 1: Inside and outside of a wire

Example 2: Solenoid



Magnetic Flux

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

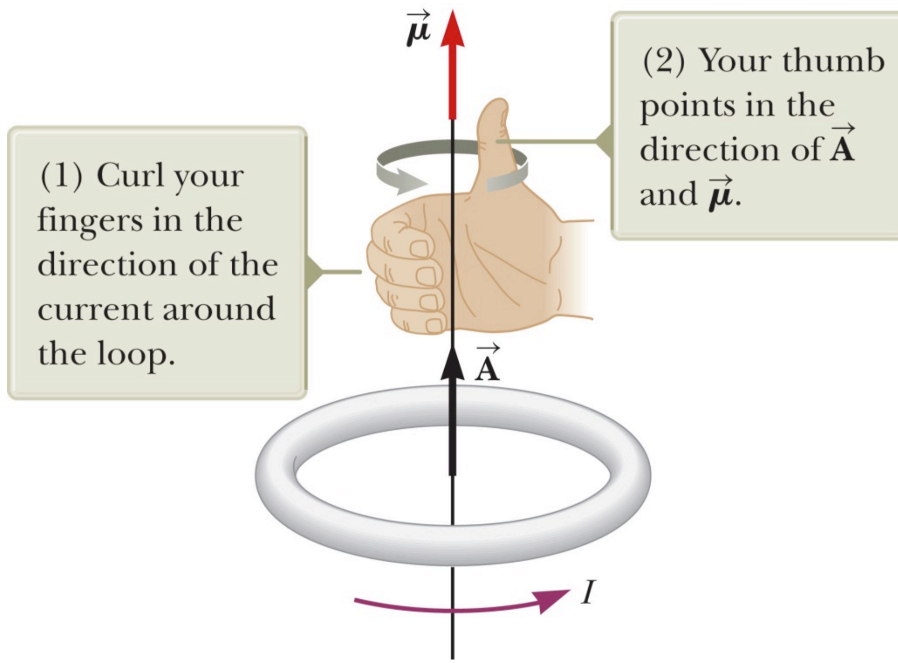


From last Thursday: Torque on current loop in magnetic field

$$\vec{\tau} = I\vec{\mathbf{A}} \times \vec{\mathbf{B}}$$

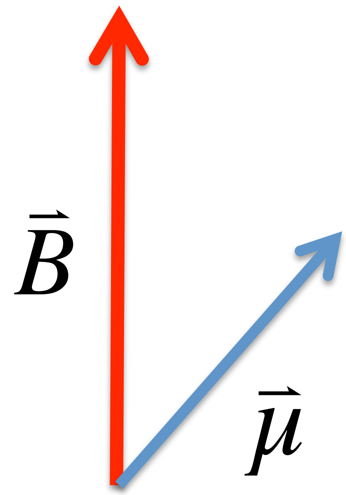
$$\vec{\mu} = I\vec{A}$$

μ : magnetic moment



Magnetic field applies torque on magnetic moment

Example: Torque on magnetic moment



Clicker question #3: Which way will the magnetic field try to orient the moment?

a. Clockwise
b. Counter clockwise
c. Out of the board
d. Into the board